

Untersuche jedes Polynom in Aufgabe **A)** und **B)** mit folgenden Schritten:

- a) Bestimme den Grad n und den Koeffizienten a_n .
- b) Führe eine Linearfaktorzerlegung durch.
- c) Untersuche das Verhalten für $x \rightarrow \pm\infty$.
- d) *Skizziere* den Verlauf des Schaubildes.

A) Linearfaktorzerlegungen von Polynomen

- | | | |
|-------------------------------------------------------|----------------------------------------------------------|-----------------------------------------------------|
| 1. $\frac{2x^4}{3} + 6x^3$ | 30. $\frac{x^3}{2} + 4x^2$ | 61. $x^2 + 3x$ |
| 2. $\frac{2x^4}{3} - \frac{22x^3}{3} + 16x^2$ | 31. $2x^2 - 4x - 30$ | 62. $2x^3 + 14x^2 - 36x$ |
| 3. $\frac{x^5}{2} + 5x^4 + \frac{9x^3}{2}$ | 32. $\frac{x^2}{4} - \frac{x}{4} - \frac{21}{2}$ | 63. $\frac{2x^3}{3} + 8x^2 + 18x$ |
| 4. $\frac{x^2}{4} + \frac{11x}{4} + \frac{15}{2}$ | 33. $\frac{x^5}{4} - \frac{3x^4}{4}$ | 64. $\frac{2x^5}{3} - 6x^3$ |
| 5. $\frac{x^2}{3} + 2x$ | 34. $\frac{x^2}{2} - x - 24$ | 65. $2x^4 + 16x^3 + 30x^2$ |
| 6. $x^2 + 2x$ | 35. $x^2 + 10x + 21$ | 66. $\frac{x^2}{4} + \frac{5x}{2} + 4$ |
| 7. $\frac{x^2}{2} + \frac{9x}{2} + 7$ | 36. $x^5 + 4x^4 - 5x^3$ | 67. $\frac{x^3}{3} + 4x^2 + 9x$ |
| 8. $\frac{x^2}{3} - \frac{5x}{3}$ | 37. $\frac{x^3}{2} - \frac{9x^2}{2} + 7x$ | 68. $\frac{x^2}{3} - \frac{11x}{3} + 6$ |
| 9. $\frac{x^3}{2} - 5x^2 + \frac{21x}{2}$ | 38. $\frac{x^2}{4} - x + \frac{3}{4}$ | 69. $\frac{x^5}{2} - 2x^4 - \frac{45x^3}{2}$ |
| 10. $\frac{x^4}{2} + \frac{5x^3}{2} - 7x^2$ | 39. $x^3 - 2x^2$ | 70. $\frac{x^2}{4} + \frac{17x}{4} + 18$ |
| 11. $\frac{x^3}{2} + 3x^2 - 8x$ | 40. $\frac{2x^2}{3} + 8x + \frac{64}{3}$ | 71. $x^2 - 9x + 8$ |
| 12. $x^2 - 10x + 16$ | 41. $2x^2 + 2x - 60$ | 72. $2x^2 + 16x - 18$ |
| 13. $\frac{2x^2}{3} - \frac{14x}{3} - 12$ | 42. $\frac{x^2}{4} + \frac{13x}{4} + 10$ | 73. $x^2 + 11x + 24$ |
| 14. $\frac{x^2}{2} + \frac{7x}{2} + 6$ | 43. $x^2 - 9x$ | 74. $2x^3 + 32x^2 + 126x$ |
| 15. $\frac{x^4}{3} + 4x^3 + \frac{32x^2}{3}$ | 44. $\frac{x^4}{4} - \frac{5x^3}{4} - \frac{7x^2}{2}$ | 75. $x^2 + 5x$ |
| 16. $x^2 + 2x + 1$ | 45. $x^3 - 4x^2 - 32x$ | 76. $\frac{x^2}{3} + \frac{7x}{3} + \frac{10}{3}$ |
| 17. $2x^2 - 18x + 28$ | 46. $2x^4 - 24x^3 + 54x^2$ | 77. $x^2 - 12x + 27$ |
| 18. $\frac{x^5}{4} + \frac{7x^4}{4} + \frac{3x^3}{2}$ | 47. $\frac{x^2}{2} - \frac{1}{2}$ | 78. $\frac{x^5}{2} + 2x^4$ |
| 19. $\frac{x^2}{2} - \frac{x}{2} - 10$ | 48. $\frac{x^4}{2} - 8x^3 + 32x^2$ | 79. $\frac{x^2}{3} - \frac{x}{3} - 2$ |
| 20. $\frac{x^2}{3} + \frac{11x}{3} + 8$ | 49. $x^2 - x - 2$ | 80. $x^5 + 13x^4 + 42x^3$ |
| 21. $\frac{x^4}{2} - x^3 - 4x^2$ | 50. $2x^2 + 16x + 30$ | 81. $2x^2 + 16x - 18$ |
| 22. $\frac{x^5}{4} + \frac{9x^4}{4} + \frac{7x^3}{2}$ | 51. $\frac{2x^3}{3} - 6x$ | 82. $\frac{2x^4}{3} - \frac{2x^2}{3}$ |
| 23. $x^5 - x^4 - 72x^3$ | 52. $\frac{x^4}{3} - \frac{8x^3}{3} + \frac{7x^2}{3}$ | 83. $\frac{x^2}{3} + \frac{10x}{3} + \frac{16}{3}$ |
| 24. $\frac{x^5}{4} + \frac{x^4}{4} - \frac{x^3}{2}$ | 53. $2x^3 + 6x^2$ | 84. $x^5 + 7x^4$ |
| 25. $\frac{x^4}{2} + \frac{3x^3}{2} - 9x^2$ | 54. $x^5 + 10x^4 + 25x^3$ | 85. $x^5 + 7x^4 - 8x^3$ |
| 26. $\frac{x^4}{4} - x^3 - 3x^2$ | 55. $\frac{x^4}{3} + 3x^3$ | 86. $x^3 - 6x^2 - 27x$ |
| 27. $\frac{2x^4}{3} + \frac{10x^3}{3} + 4x^2$ | 56. $\frac{x^4}{4} - \frac{7x^3}{4}$ | 87. $\frac{x^5}{4} - \frac{x^4}{2} + \frac{x^3}{4}$ |
| 28. $\frac{x^2}{3} - \frac{13x}{3} + 14$ | 57. $x^2 - 6x$ | 88. $\frac{2x^3}{3} + 6x^2 + \frac{40x}{3}$ |
| 29. $\frac{x^4}{2} + x^3$ | 58. $\frac{x^2}{2} - \frac{7x}{2} + 3$ | 89. $\frac{x^5}{3} + \frac{7x^4}{3} + 2x^3$ |
| | 59. $\frac{x^2}{4} - \frac{3x}{4}$ | 90. $\frac{2x^2}{3} - \frac{4x}{3} - 16$ |
| | 60. $\frac{2x^4}{3} + \frac{22x^3}{3} + \frac{56x^2}{3}$ | |

B) Linearfaktorzerlegungen von Polynomen - mit Substitution

- | | | |
|-------------------------------|-----------------------------|-------------------------------|
| 1. $4x^7 - 136x^5 + 900x^3$ | 5. $x^6 - 29x^4 + 100x^2$ | 9. $2x^7 - 82x^5 + 800x^3$ |
| 2. $6x^7 - 12x^5 + 6x^3$ | 6. $7x^7 - 126x^5 + 567x^3$ | 10. $3x^6 - 78x^4 + 75x^2$ |
| 3. $6x^6 - 588x^4 + 14406x^2$ | 7. $3x^6 - 159x^4 + 588x^2$ | 11. $3x^7 - 195x^5 + 2352x^3$ |
| 4. $7x^6 - 224x^4 + 1792x^2$ | 8. $3x^6 - 87x^4 + 300x^2$ | 12. $6x^6 - 300x^4 + 294x^2$ |

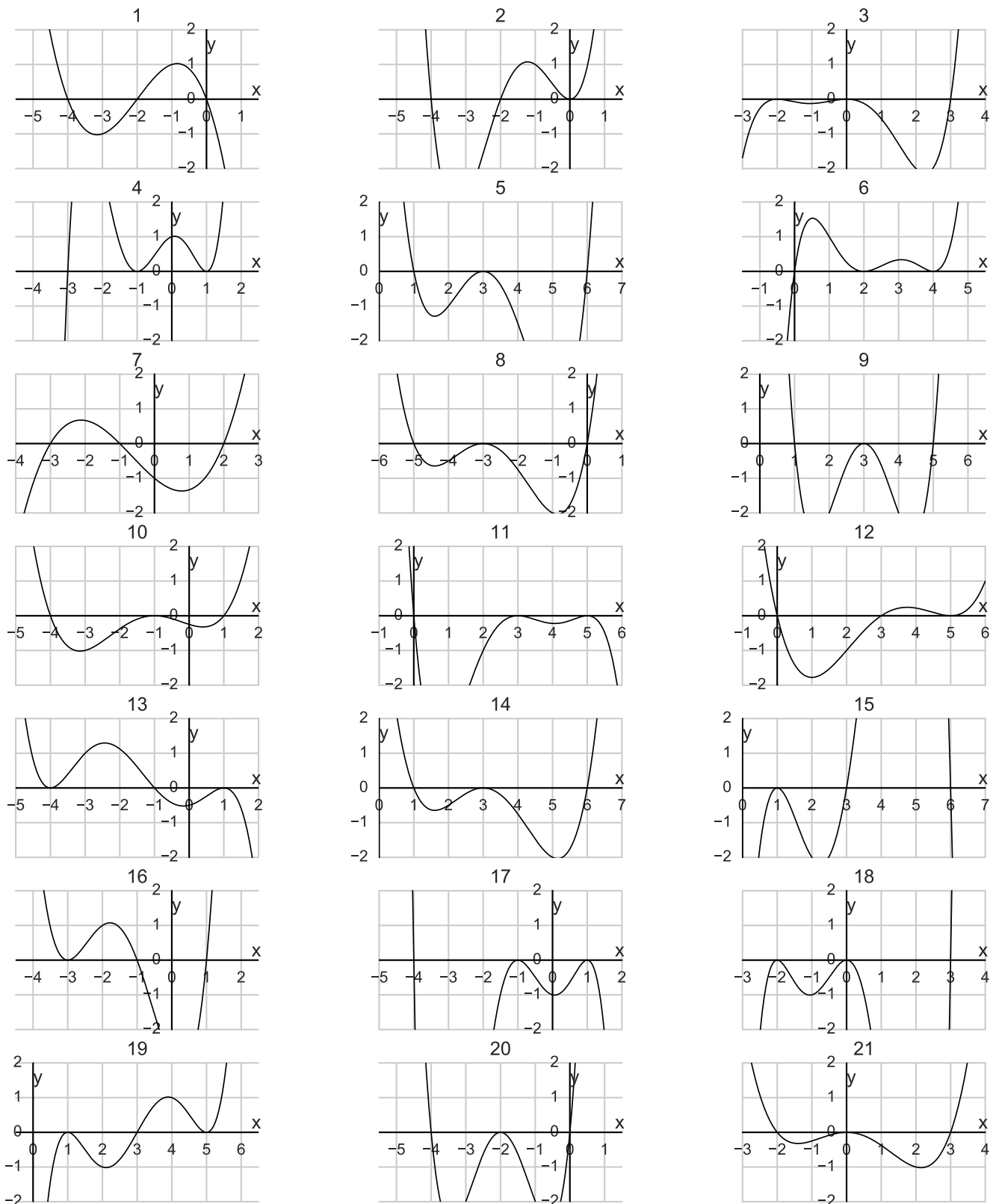
A) Linearfaktorzerlegungen von Polynomen (Zerlegung durchgeführt)

1. $\frac{2x^4}{3} + 6x^3 = \frac{2x^3}{3}(x+9)$
2. $\frac{2x^4}{3} - \frac{22x^3}{3} + 16x^2 = \frac{2x^2}{3}(x-8)(x-3)$
3. $\frac{x^5}{2} + 5x^4 + \frac{9x^3}{2} = \frac{x^3}{2}(x+1)(x+9)$
4. $\frac{x^2}{4} + \frac{11x}{4} + \frac{15}{2} = \frac{1}{4}(x+5)(x+6)$
5. $\frac{x^2}{3} + 2x = \frac{x}{3}(x+6)$
6. $x^2 + 2x = x(x+2)$
7. $\frac{x^2}{2} + \frac{9x}{2} + 7 = \frac{1}{2}(x+2)(x+7)$
8. $\frac{x^2}{3} - \frac{5x}{3} = \frac{x}{3}(x-5)$
9. $\frac{x^2}{2} - 5x^2 + \frac{21x}{2} = \frac{x}{2}(x-7)(x-3)$
10. $\frac{x^4}{2} + \frac{5x^3}{2} - 7x^2 = \frac{x^2}{2}(x-2)(x+7)$
11. $\frac{x^3}{2} + 3x^2 - 8x = \frac{x}{2}(x-2)(x+8)$
12. $x^2 - 10x + 16 = (x-8)(x-2)$
13. $\frac{2x^2}{3} - \frac{14x}{3} - 12 = \frac{2}{3}(x-9)(x+2)$
14. $\frac{x^2}{2} + \frac{7x}{2} + 6 = \frac{1}{2}(x+3)(x+4)$
15. $\frac{x^4}{3} + 4x^3 + \frac{32x^2}{3} = \frac{x^2}{3}(x+4)(x+8)$
16. $x^2 + 2x + 1 = (x+1)^2$
17. $2x^2 - 18x + 28 = 2(x-7)(x-2)$
18. $\frac{x^5}{4} + \frac{7x^4}{4} + \frac{3x^3}{2} = \frac{x^3}{4}(x+1)(x+6)$
19. $\frac{x^2}{2} - \frac{x}{2} - 10 = \frac{1}{2}(x-5)(x+4)$
20. $\frac{x^2}{3} + \frac{11x}{3} + 8 = \frac{1}{3}(x+3)(x+8)$
21. $\frac{x^4}{2} - x^3 - 4x^2 = \frac{x^2}{2}(x-4)(x+2)$
22. $\frac{x^5}{4} + \frac{9x^4}{4} + \frac{7x^3}{2} = \frac{x^3}{4}(x+2)(x+7)$
23. $x^5 - x^4 - 72x^3 = x^3(x-9)(x+8)$
24. $\frac{x^5}{4} + \frac{x^4}{4} - \frac{x^3}{2} = \frac{x^3}{4}(x-1)(x+2)$
25. $\frac{x^4}{2} + \frac{3x^3}{2} - 9x^2 = \frac{x^2}{2}(x-3)(x+6)$
26. $\frac{x^4}{4} - x^3 - 3x^2 = \frac{x^2}{4}(x-6)(x+2)$
27. $\frac{2x^4}{3} + \frac{10x^3}{3} + 4x^2 = \frac{2x^2}{3}(x+2)(x+3)$
28. $\frac{x^3}{3} - \frac{13x}{3} + 14 = \frac{1}{3}(x-7)(x-6)$
29. $\frac{x^4}{2} + x^3 = \frac{x^3}{2}(x+2)$
30. $\frac{x^3}{2} + 4x^2 = \frac{x^2}{2}(x+8)$
31. $2x^2 - 4x - 30 = 2(x-5)(x+3)$
32. $\frac{x^2}{4} - \frac{x}{4} - \frac{21}{2} = \frac{1}{4}(x-7)(x+6)$
33. $\frac{x^5}{4} - \frac{3x^4}{4} = \frac{x^4}{4}(x-3)$
34. $\frac{x^2}{2} - x - 24 = \frac{1}{2}(x-8)(x+6)$
35. $x^2 + 10x + 21 = (x+3)(x+7)$
36. $x^5 + 4x^4 - 5x^3 = x^3(x-1)(x+5)$
37. $\frac{x^3}{2} - \frac{9x^2}{2} + 7x = \frac{x}{2}(x-7)(x-2)$
38. $\frac{x^2}{4} - x + \frac{3}{4} = \frac{1}{4}(x-3)(x-1)$
39. $x^3 - 2x^2 = x^2(x-2)$
40. $\frac{2x^2}{3} + 8x + \frac{64}{3} = \frac{2}{3}(x+4)(x+8)$
41. $2x^2 + 2x - 60 = 2(x-5)(x+6)$
42. $\frac{x^2}{4} + \frac{13x}{4} + 10 = \frac{1}{4}(x+5)(x+8)$
43. $x^2 - 9x = x(x-9)$
44. $\frac{x^4}{4} - \frac{5x^3}{4} - \frac{7x^2}{2} = \frac{x^2}{4}(x-7)(x+2)$
45. $x^3 - 4x^2 - 32x = x(x-8)(x+4)$
46. $2x^4 - 24x^3 + 54x^2 = 2x^2(x-9)(x-3)$
47. $\frac{x^2}{2} - \frac{1}{2} = \frac{1}{2}(x-1)(x+1)$
48. $\frac{x^4}{2} - 8x^3 + 32x^2 = \frac{x^2}{2}(x-8)^2$
49. $x^2 - x - 2 = (x-2)(x+1)$
50. $2x^2 + 16x + 30 = 2(x+3)(x+5)$
51. $\frac{2x^3}{3} - 6x = \frac{2x}{3}(x-3)(x+3)$
52. $\frac{x^4}{3} - \frac{8x^3}{3} + \frac{7x^2}{3} = \frac{x^2}{3}(x-7)(x-1)$
53. $2x^3 + 6x^2 = 2x^2(x+3)$
54. $x^5 + 10x^4 + 25x^3 = x^3(x+5)^2$
55. $\frac{x^4}{3} + 3x^3 = \frac{x^3}{3}(x+9)$
56. $\frac{x^4}{4} - \frac{7x^3}{4} = \frac{x^3}{4}(x-7)$
57. $x^2 - 6x = x(x-6)$
58. $\frac{x^2}{2} - \frac{7x}{2} + 3 = \frac{1}{2}(x-6)(x-1)$
59. $\frac{x^2}{4} - \frac{3x}{4} = \frac{x}{4}(x-3)$
60. $\frac{2x^4}{3} + \frac{22x^3}{3} + \frac{56x^2}{3} = \frac{2x^2}{3}(x+4)(x+7)$
61. $x^2 + 3x = x(x+3)$
62. $2x^3 + 14x^2 - 36x = 2x(x-2)(x+9)$
63. $\frac{2x^3}{3} + 8x^2 + 18x = \frac{2x}{3}(x+3)(x+9)$
64. $\frac{2x^5}{3} - 6x^3 = \frac{2x^3}{3}(x-3)(x+3)$
65. $2x^4 + 16x^3 + 30x^2 = 2x^2(x+3)(x+5)$
66. $\frac{x^2}{4} + \frac{5x}{2} + 4 = \frac{1}{4}(x+2)(x+8)$
67. $\frac{x^3}{3} + 4x^2 + 9x = \frac{x}{3}(x+3)(x+9)$
68. $\frac{x^2}{3} - \frac{11x}{3} + 6 = \frac{1}{3}(x-9)(x-2)$
69. $\frac{x^5}{2} - 2x^4 - \frac{45x^3}{2} = \frac{x^3}{2}(x-9)(x+5)$
70. $\frac{x^2}{4} + \frac{17x}{4} + 18 = \frac{1}{4}(x+8)(x+9)$
71. $x^2 - 9x + 8 = (x-8)(x-1)$
72. $2x^2 + 16x - 18 = 2(x-1)(x+9)$
73. $x^2 + 11x + 24 = (x+3)(x+8)$
74. $2x^3 + 32x^2 + 126x = 2x(x+7)(x+9)$
75. $x^2 + 5x = x(x+5)$
76. $\frac{x^2}{3} + \frac{7x}{3} + \frac{10}{3} = \frac{1}{3}(x+2)(x+5)$
77. $x^2 - 12x + 27 = (x-9)(x-3)$
78. $\frac{x^5}{2} + 2x^4 = \frac{x^4}{2}(x+4)$
79. $\frac{x^2}{3} - \frac{x}{3} - 2 = \frac{1}{3}(x-3)(x+2)$
80. $x^5 + 13x^4 + 42x^3 = x^3(x+6)(x+7)$
81. $2x^2 + 16x - 18 = 2(x-1)(x+9)$
82. $\frac{2x^4}{3} - \frac{2x^2}{3} = \frac{2x^2}{3}(x-1)(x+1)$
83. $\frac{x^3}{3} + \frac{10x}{3} + \frac{16}{3} = \frac{1}{3}(x+2)(x+8)$
84. $x^5 + 7x^4 = x^4(x+7)$
85. $x^5 + 7x^4 - 8x^3 = x^3(x-1)(x+8)$
86. $x^3 - 6x^2 - 27x = x(x-9)(x+3)$
87. $\frac{x^5}{4} - \frac{x^4}{2} + \frac{x^3}{4} = \frac{x^3}{4}(x-1)^2$
88. $\frac{2x^3}{3} + 6x^2 + \frac{40x}{3} = \frac{2x}{3}(x+4)(x+5)$
89. $\frac{x^5}{3} + \frac{7x^4}{3} + 2x^3 = \frac{x^3}{3}(x+1)(x+6)$
90. $\frac{2x^2}{3} - \frac{4x}{3} - 16 = \frac{2}{3}(x-6)(x+4)$

B) Linearfaktorzerlegungen von Polynomen - mit Substitution (Zerlegung durchgeführt)

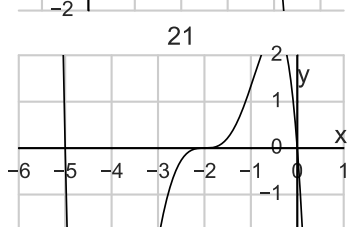
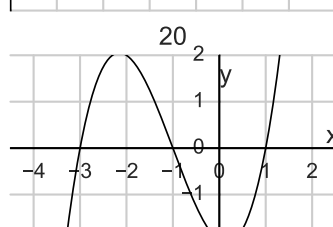
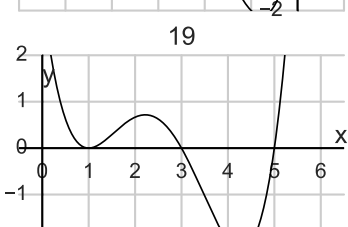
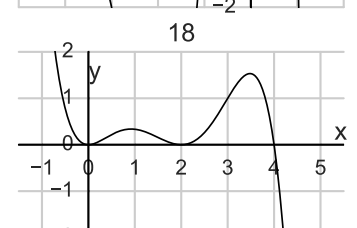
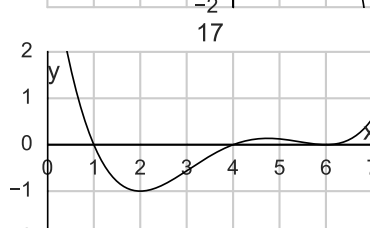
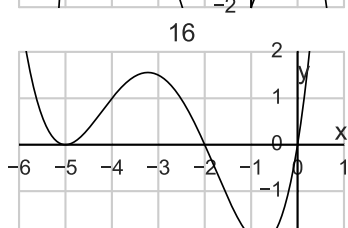
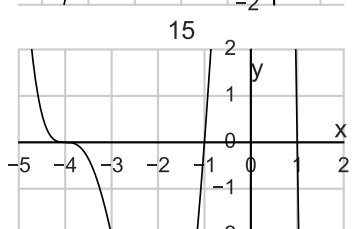
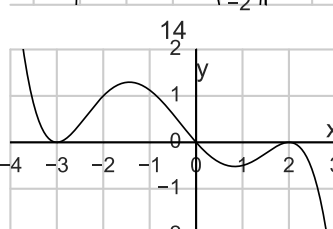
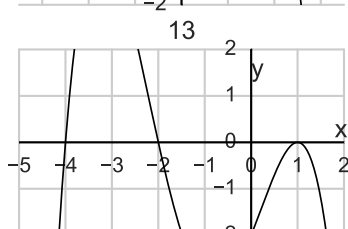
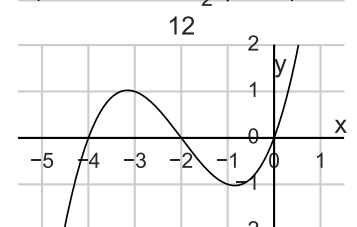
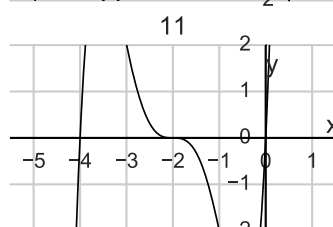
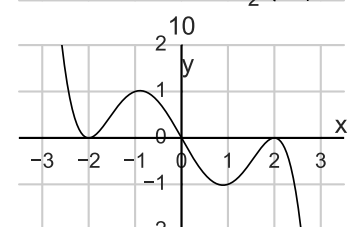
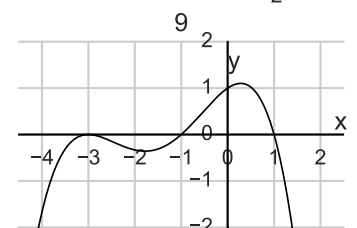
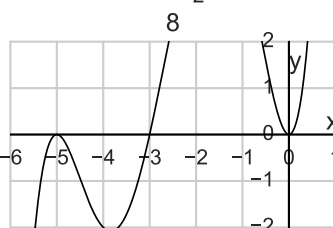
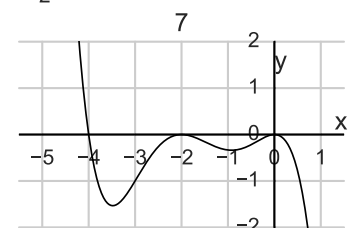
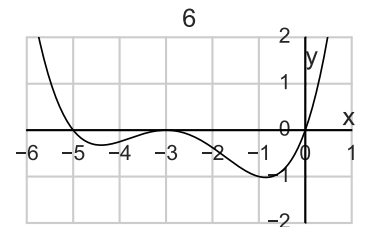
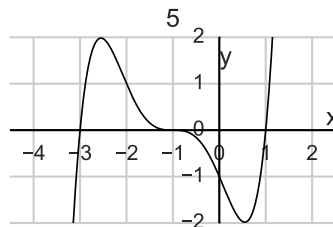
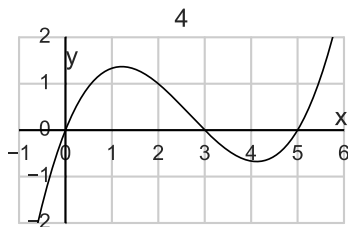
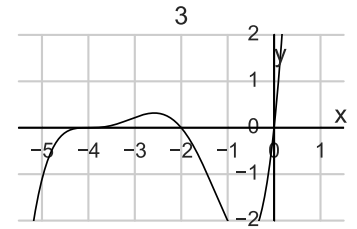
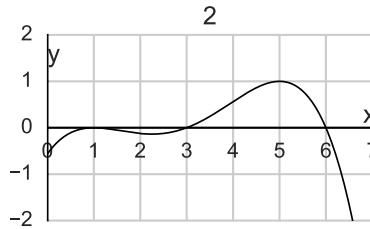
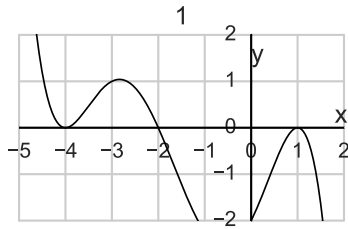
1. $4x^7 - 136x^5 + 900x^3 = 4x^3(x-5)(x-3)(x+3)(x+5)$
2. $6x^7 - 12x^5 + 6x^3 = 6x^3(x-1)^2(x+1)^2$
3. $6x^6 - 588x^4 + 14406x^2 = 6x^2(x-7)^2(x+7)^2$
4. $7x^6 - 224x^4 + 1792x^2 = 7x^2(x-4)^2(x+4)^2$
5. $x^6 - 29x^4 + 100x^2 = x^2(x-5)(x-2)(x+2)(x+5)$
6. $7x^7 - 126x^5 + 567x^3 = 7x^3(x-3)^2(x+3)^2$
7. $3x^6 - 159x^4 + 588x^2 = 3x^2(x-7)(x-2)(x+2)(x+7)$
8. $3x^6 - 87x^4 + 300x^2 = 3x^2(x-5)(x-2)(x+2)(x+5)$
9. $2x^7 - 82x^5 + 800x^3 = 2x^3(x-5)(x-4)(x+4)(x+5)$
10. $3x^6 - 78x^4 + 75x^2 = 3x^2(x-5)(x-1)(x+1)(x+5)$
11. $3x^7 - 195x^5 + 2352x^3 = 3x^3(x-7)(x-4)(x+4)(x+7)$
12. $6x^6 - 300x^4 + 294x^2 = 6x^2(x-7)(x-1)(x+1)(x+7)$

C) Funktionstermbestimmungen aus Schaubildern: einfache und doppelte Nullstellen



- 1. $-\frac{3}{2}(x+2)(x+4)$
- 2. $\frac{3}{2}(x+2)(x+4)$
- 3. $\frac{3x^2}{2}(x-3)(x+2)$
- 4. $\frac{3}{4}(x-1)^2(x+1)^2(x+3)$
- 5. $\frac{1}{4}(x-6)(x-3)^2(x-1)$
- 6. $\frac{1}{2}(x-4)^2(x-2)$
- 7. $\frac{6}{9}(x-2)(x+1)(x+3)$
- 8. $\frac{1}{2}(x+3)^2(x+5)$
- 9. $\frac{3}{2}(x-5)(x-3)^2(x-1)$
- 10. $\frac{16}{1}(x-1)(x+1)^2(x+4)$
- 11. $-\frac{18}{x}(x-5)^2(x-3)^2$
- 12. $\frac{18}{x}(x-5)^2(x-3)$
- 13. $-\frac{32}{1}(x-1)^2(x+1)(x+4)^2$
- 14. $\frac{8}{1}(x-6)(x-3)^2(x-1)$
- 15. $-\frac{1}{1}(x-6)(x-3)(x-1)^2$
- 16. $\frac{3}{1}(x-1)(x+1)(x+3)^2$
- 17. $-\frac{1}{4}(x-1)^2(x+1)^2(x+4)$
- 18. $\frac{4}{x^2}(x-3)(x+2)^2$
- 19. $\frac{9}{x}(x-5)^2(x-3)(x-1)^2$
- 20. $\frac{2}{2x}(x+2)^2(x+4)$
- 21. $\frac{16}{x^2}(x-3)(x+2)$

D) Funktionstermbestimmungen aus Schaubildern: einfache, doppelte und dreifache Nullstellen



- 21. $-\frac{3}{x}(x+2)^3(x+5)$
- 20. $\frac{3}{2}(x-1)(x+1)(x+3)$
- 19. $(x-5)(x-3)(x-1)^2$
- 18. $-\frac{9}{x^2}(x-4)(x-2)^2$
- 17. $\frac{32}{x}(x-6)^2(x-4)(x-1)$
- 16. $(x+2)(x+5)^2$
- 15. $-\frac{1}{4}(x-1)(x+1)(x+4)^3$

- 14. $-\frac{32}{x}(x-2)^2(x+3)^2$
- 13. $-\frac{1}{4}(x-1)^2(x+2)(x+4)$
- 12. $\frac{3}{x}(x+2)(x+4)$
- 11. $\frac{2}{x}(x+2)^3(x+4)$
- 10. $-\frac{6}{x}(x-2)^2(x+2)^2$
- 9. $-\frac{6}{x}(x-1)(x+1)(x+3)^2$
- 8. $\frac{8}{x^2}(x+3)(x+5)^2$

- 7. $-\frac{6}{x^2}(x+2)^2(x+4)$
- 6. $\frac{16}{x}(x+3)^2(x+5)$
- 5. $\frac{3}{x}(x-1)(x+1)^3(x+3)$
- 4. $\frac{6}{x}(x-5)(x-3)$
- 3. $\frac{2}{x}(x+2)(x+4)^3$
- 2. $-\frac{32}{x}(x-6)(x-3)(x-1)^2$
- 1. $-\frac{16}{x}(x-1)^2(x+2)(x+4)^2$